

three months. The electromotive forces at six temperatures are given in Table I. Figure 1 shows the variation in electromotive force in one series of cells as the bismuth content of the ternary amalgam varied. The curves for the other series were similar. In Fig. 2 the compositions of the amalgams having the same cadmium activity at 25° are plotted. The dotted line is the approximate boundary of the region of constant cadmium activity.

The region of constant cadmium activity is sufficiently large to allow easy reproducibility of electromotive force in cells in which the ternary amalgam is used, and to prevent any probable

changes in composition from affecting the electromotive force. An amalgam of a composition in the neighborhood of 11% Cd, 15% Bi and 74% Hg, which is shown in Fig. 2 by a black dot, should be satisfactory for use in modified Weston cells. This composition is well within the constant-activity region from 10 to 40°.

Summary

An investigation has been made of the composition limits for cadmium-bismuth amalgams within which the cadmium activity is independent of the composition between 10 and 40°.

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Measurements of the Viscosity of Liquid Helium II

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In connection with the design of apparatus for adiabatic demagnetization investigations at low temperatures, we became interested in the viscosity of liquid helium near 1°K. Moreover, several investigators have recently reported experiments which show that liquid helium II is a substance of great interest. Wilhelm, Misener and Clark¹ have measured the relative viscosity of liquid helium from 4.2 to 2.2°K. They found that the viscosity increased normally with decreasing temperature until the region of the so-called "lambda point" was reached. At about 2.2° the viscosity decreased rapidly.

Burton² used the above measurements as a basis for the calculation of absolute values of the viscosity, η . At the boiling point he found $\eta = 1.1 \times 10^{-4}$ poise. At 2.3°K. it increased to 2.7×10^{-4} . It then dropped to 3.3×10^{-5} poise at 2.2°K.

More recently Kapitza³ and Allen and Misener⁴ have reported that at lower temperatures the viscosity decreased to such a phenomenally low value that it could not be determined from their experiments. Allen and Misener used a fine capillary of thermometer tubing and Kapitza experimented with optically plane plates in contact with each other. In each case flow was produced by a small head of liquid helium. The

above workers concluded that even under the above conditions the flow was turbulent. Kapitza estimated that the viscosity below the "lambda point" was less than 10^{-9} poise. Allen and Misener, whose experiments were made at the temperatures 1.07 and 2.17°K., estimated 4×10^{-9} poise as the upper limit of viscosity in this region. These conclusions are not in agreement with measurements of the present authors⁵ to be described below. Kapitza considered the flow to be turbulent, with a Reynolds number of 750,000, in his experiment. Allen and Misener stated that the flow in their experiments was neither ordinary laminar nor turbulent and suggested the possibility of slip at the surface.

It has since become evident that the conclusions quoted above are not valid due to misinterpretation as to what actually occurred in the experiments.

Kürti, Rollin and Simon⁶ have suggested that the seemingly very high thermal conductivity along a tube leading to a chamber in which liquid helium is being evaporated under reduced pressure is due to a film on the tube surface. We have enjoyed the opportunity of discussing this question with Dr. B. V. Rollin on several occasions. Some of our adiabatic demagnetization experiments have been hindered by effects of this

(1) Wilhelm, Misener and Clark, *Proc. Roy. Soc. (London)*, **A151**, 342 (1935).

(2) Burton, *Nature*, **135**, 265 (1935).

(3) Kapitza, *ibid.*, **141**, 74 (1938).

(4) Allen and Misener, *ibid.*, **141**, 75 (1938).

(5) Giauque, Stout and Barieau, *Phys. Rev.*, **54**, 147 (1938).

(6) (a) Kürti, Rollin and Simon, *Physica*, **3**, 266 (1936); (b) Rollin, *Actes 7th Cong. int. du Froid*, **1**, 187 (1936).

kind and all of our observations are very satisfactorily explained by the transfer of helium up the walls of the tube until it reached the warmer upper portions of the apparatus. Rollin^{6b} also called attention to an experiment of Kamerlingh Onnes⁷ in which he showed that when one Dewar vessel of liquid helium was partially immersed in another the levels rapidly equalized when the system was under reduced pressure. The presence of a surface film readily accounts for this effect. The above experiment has recently been repeated with a partially submerged or elevated beaker by Daunt and Mendelssohn,⁸ who obtained a similar result with liquid helium II. They report that the rate of transfer increases rapidly with decreasing temperature. Kikoin and Lasarew⁹ have extended Rollin's experiments with liquid helium films on the walls of tubes. From measurements with a heater located in a tube, and about 15 cm. above the bath, they estimate the film to be about 10^{-6} cm. thick.

In view of the large amount of evidence that rapid transfer of liquid helium II occurs through the mechanism of surface films on various substances, it became rather obvious⁵ that the large rates of flow observed in experiments with capillaries, whose surfaces and ends are accessible to the bath, are due principally to flow on the outer surface of the apparatus rather than within the capillary channels. We were forced to conclude that none of the above experiments permitted even an estimate of the viscosity of liquid helium below 2.2°K .

Since the completion of our experiments Keesom and MacWood¹⁰ have published the results of a series of measurements by an oscillating disk method which is not open to the objection pointed out above. These authors find that the viscosity decreases with temperature in contradiction to the results of Wilhelm, Misener and Clark,¹ who found a marked increase with decreasing temperature in the region above the "lambda point."

Also our attention has been called to a footnote in which Kapitza¹¹ states that unpublished measurements from 2.19 to 4.22°K . lead to a constant viscosity, $\eta = 1.1 \times 10^{-5} \approx 0.05$ poise. No details of the measurements are given.

Our results above 2.19°K . agree most closely

(7) Kamerlingh Onnes, *Comm. Phys. Lab. Leiden*, No. 159 (1922); *Trans. Faraday Soc.*, **18**, Part 2, No. 53 (1922).

(8) Daunt and Mendelssohn, *Nature*, **141**, 911 (1938).

(9) Kikoin and Lasarew, *ibid.*, **141**, 912 (1938); **142**, 289 (1938).

(10) Keesom and MacWood, *Physica*, **5**, 737 (1938).

(11) Kapitza, *Compt. rend. acad. sci. U. R. S. S.*, **18**, 21 (1938).

with those of Keesom and MacWood. Below 2.0°K . the viscosity values obtained in the present experiment are smaller than those of Keesom and MacWood by a factor ranging from 10 at 2° to 22 at 1.47°K .

Our first opportunity to measure the viscosity of liquid helium presented itself rather fortuitously. An experiment of another sort involved the use of a Pyrex glass tube with two tungsten leads sealed into it. The apparatus was attached to a high vacuum system. It was tight at ordinary temperatures both before and after our experiments. However, at helium temperatures a considerable leak developed. When the pressure on the bath was reduced the amount of the leak decreased until the temperature region of helium II was reached. As the temperature decreased below about 2.2°K . the amount of helium flowing through the leak greatly increased. It was evident that we were dealing with a capillary flow which was largely controlled by the properties of the liquid. As a matter of fact we had observed this effect on one or two previous occasions and had explained it as a rather surprising effect of pressure change on a fissure. It is now evident that immersing an evacuated apparatus in liquid helium II is perhaps the most severe test of tightness to which it can be subjected. Not only does the liquid possess a lower viscosity than the very low value of helium gas at corresponding temperatures but the liquid, after evaporation within the apparatus, forms relatively large amounts of gas.

Following a series of measurements on the above leak which will be referred to as capillary A, we prepared a capillary by a method which allowed a fairly good knowledge of the geometry of the passage. A glass capillary was filled with solder and the differential contraction with temperature was used to produce a small channel between the solder and the glass. This tube will be referred to as capillary B.

As we were interested in the tightness of tungsten seals in liquid helium II, three seals were prepared. Two were wires 0.1 cm. in diameter sealed through 1 cm. of Pyrex glass G702EJ and one wire 0.1 cm. in diameter sealed through 1 cm. of nonex glass G702P which is designed to match the coefficient of expansion of tungsten. No leak was detected in any of these seals when they were immersed in liquid helium II. Although they were tested about three days after the seals were made,

previous experience gives us some ground for suspecting that failure might occur at some later time.

Theory of the Method.—In the present experiments the entrance of the capillary was immersed in liquid helium and the exit opened into a line attached to a gas pumping system. At the higher temperatures the liquid entered the channel, evaporated at some point and emerged as a gas. When the capillaries and rates of flow are as small as those used in this work the process may be considered to be isothermal.

The penetration of the liquid into the capillary depends on the action of surface tension, vapor pressure, hydrostatic pressure and the viscosities of both gas and liquid. At the boiling point it may be shown from equations and data to be given later that the liquid penetrates into capillary B to the extent of about 7% of its length. Applying a suitable correction for the penetration of the liquid, the constant of the capillary can be evaluated in terms of the known viscosity of helium gas, the observed flow and the pressure difference.

At lower temperatures the liquid advances far into the capillary until, when the dimensions are suitable, the channel is practically filled with liquid at the temperature of the "lambda point." With capillary B of the present experiment the channel was about 96% full of liquid at 2.26°K. and liquid was flowing from the exit at all temperatures below 1.8°K.

Although it will be shown that an accurate knowledge of the tube geometry is not essential for most of the results presented here, it is of interest to consider this matter in some detail. When a glass capillary is filled with some material such as solder, with a higher coefficient of expansion than that of glass, a channel will be formed when the apparatus is cooled. The form of the channel must be either annular, when the solder is central, crescent, when the solder touches one side, or some intermediate case. It will not be necessary to consider more than the limiting cases for our present purposes.

The ratio of channel width to its circumference is such that the annular form may be considered to correspond to the well-known case for parallel plates.¹² The average velocity

$$\bar{v} = \frac{(D-d)^2 \Delta P}{48\eta L} \quad (1)$$

(12) E. g. Barr, "A Monograph of Viscosity," Oxford University Press, London, 1931.

where η is the viscosity, ΔP is the pressure drop spent on viscous friction, $(D-d)$ the difference between the diameters of glass and solder, respectively, and L is the length of the capillary

$$\text{The volume/sec.} = V = \frac{\pi D(D-d)^3 \Delta P}{96\eta L} \quad (2)$$

$$\text{The mass/sec.} = m = \frac{\pi D(D-d)^3 \Delta P \rho}{96\eta L} \quad (3)$$

where ρ is the density.

$$\text{The kinetic energy of flow/g.} = \text{K. E.} = \frac{27}{35} \bar{v}^2 \quad (4)$$

Let $\Delta P_{\text{total}} = \Delta P + \Delta P_{\text{K.E.}}$ where $\Delta P_{\text{K.E.}}$ is the pressure drop used for the kinetic energy of flow. Then for liquid flow

$$\eta = \frac{\pi D(D-d)^3 \rho}{96Lm} \left[\Delta P_{\text{total}} - \frac{108m^2}{35\pi^2 D^2 (D-d)^2 \rho} \right] \quad (5)$$

For isothermal gas flow Bernoulli's theorem gives Available work = Frictional work + K. E.

$$-VdP = \frac{96\eta_0 V^2 dL}{\pi D(D-d)^3} + \frac{216m V dV}{35\pi^2 D^2 (D-d)^2} \quad (6)$$

$$-\int_{P_1}^{P_2} \frac{dP}{V} = \frac{96\eta_0}{\pi D(D-d)^3} \int_0^L dL + \frac{216m}{35\pi^2 D^2 (D-d)^2} \int_{V_1}^{V_2} \frac{dV}{V} \quad (7)$$

For an ideal gas

$$V = \frac{mRT}{MP}$$

where M is the molecular weight.

$$\frac{M(P_1^2 - P_2^2)}{2mRT} = \frac{96L\eta_0}{\pi D(D-d)^3} + \frac{216m \ln(V_2/V_1)}{35\pi^2 D^2 (D-d)^2} \quad (8)$$

However, helium gas is very imperfect near its boiling point and a correction is desirable. It will be sufficiently accurate to use an equation of state of the form

$$PV = \frac{mRT}{M} (1 + \lambda P) \quad (9)$$

In this case Equation 8 becomes

$$\frac{M}{mRT\lambda^2} \left[\lambda(P_1 - P_2) - \ln \frac{1 + \lambda P_1}{1 + \lambda P_2} \right] = \frac{96L\eta_0}{\pi D(D-d)^3} + \frac{216m \ln(V_2/V_1)}{35\pi^2 D^2 (D-d)^2} \quad (10)$$

When $(D-d)/d$ is small the following approximations are valid for flow through a crescent-shaped passage

$$\bar{v} = \frac{5(D-d)^2 \Delta P}{96\eta L} \quad (11)$$

$$V = \frac{5\pi D(D-d)^3 \Delta P}{192\eta L} \quad (12)$$

$$m = \frac{5\pi D(D-d)^3 \Delta P \rho}{192\eta L} \quad (13)$$

$$\text{K. E.} = \frac{11583}{8750} \bar{v}^2 \quad (14)$$

$$\eta = \frac{5\pi D(D-d)^3 \rho}{192Lm} \left[\Delta P_{\text{total}} - \frac{23166 m^2}{4375\pi^2 D^2 (D-d)^2 \rho} \right] \quad (15)$$

The viscosity of the gas is related to the capillary constants by an equation corresponding to Equation 10. In a capillary of crescent shaped section

$$\frac{M}{mRT\lambda^2} \left[\lambda(P_1 - P_2) - \ln \frac{1 + \lambda P_1}{1 + \lambda P_2} \right] = \frac{192L\eta_0}{5\pi D(D-d)^3} + \frac{46332m \ln(V_2/V_1)}{4375\pi^2 D^2(D-d)^2} \quad (16)$$

It is interesting and important to note that calculation showed that the last term in Equations 8, 10 and 16 may be entirely neglected even when the kinetic energy at the exit is not negligible in comparison with the frictional work. At the boiling point of helium in the present experiments the last term of Equation 10 was less than 0.01% of the other two terms.

In the case of a liquid the kinetic energy and viscous friction work terms retain their relative importance in the final equation; however, the kinetic energy terms in Equations 5 and 15 are quite negligible in the present work. In the worst case of the experiments with capillary B the correction for the kinetic energy of the liquid was less than 0.02%. The above circumstances are especially fortunate since they make a distinction between the various possible capillary geometries of little importance in the present work.

Thus when the capillary contains only gas

$$\frac{M}{mRT\lambda^2} \left[\lambda(P_1 - P_2) - \ln \frac{1 + \lambda P_1}{1 + \lambda P_2} \right] = \frac{L}{K} \eta_0 \quad (17)$$

and when the capillary is filled with liquid

$$\frac{\Delta P \rho}{m} = \frac{L}{K} \eta_l \quad (18)$$

where L/K is the same constant in each equation.

The Experimental Observations.—In the case of the observations on capillary A the helium flow was obtained by observing the pressure drop due to flow along a glass line at room temperature. Rates of flow corresponding to different pressures were determined later. Measured pressure due to flow at the boiling point was too small for precise measurement and the results at lower temperatures with capillary A are perhaps subject to a possible error of 20% due to the magnitude of this one calibrating pressure. As has been mentioned above this experiment was performed on the spur of the moment due to the accidental presence of a channel of suitable magnitude for viscosity measurements and in the following experiments this error was largely eliminated. In the measurements with capillary B, not only was the flow pressure, corresponding to the boiling point, of suit-

able magnitude but this method of observation was used only as a check on the results. The actual data were obtained from volumetric measurements.

To obtain the exit and line pressures the observed pressures were corrected for the effect of thermomolecular pressure using the data of Weber, Keesom and Schmidt¹³; the greater part of the corrections were made from the experimental curve given by Weber, Keesom and Schmidt and Equation VII of Weber and Schmidt^{13b} was used for the very low concentrations. The line pressures were also corrected for the effect of flow. This was done by calibration of all the line used at temperatures above that of liquid air and calculating the effect in the straight 3 mm. line between the temperatures of liquid air and liquid helium. As the low temperature portion of the line was included in the volume measuring system the above pressure calculations were of most importance in determining the amounts of gas. About 10% of the effective line volume (*i. e.*, in terms of volume/temperature) was at low temperatures in the worst case. Small inaccuracies in the line temperatures in the gradient between helium and liquid air temperatures constitute what is probably the greatest experimental error. This could be avoided by having the pumping system deliver the gas to a constant temperature measuring system.

The applied pressure consisted in part of the hydrostatic pressure of the liquid helium bath. In order to observe the liquid level a piece of constantan wire had been run throughout the length of the liquid helium dewar. At intervals small pieces of copper and copper lead wires were attached. The whole apparatus was placed within a long solenoid magnet which was available. When an alternating field was applied eddy current heating in the pieces of copper showed which junctions were above liquid level. Thus the time at which the surface passed a given junction could be determined. The thermocouples were located beside the glass helium lines and also served to give temperatures needed in estimating amounts of gas in the volumetric measurements.

A most interesting feature of the above thermocouple arrangement was that it failed to work when the bath was in the temperature region of liquid helium II. Evidently a surface film of

(13) (a) Weber, Keesom and Schmidt, *Comm. Phys. Lab. Leiden*, No. 246a; (b) Weber and Schmidt, *ibid.*, No. 246c.

TABLE I
DATA FOR CALCULATING THE VISCOSITY OF LIQUID HELIUM II
Capillary A

P_v represents the vapor pressure, P_h the hydrostatic pressure and P the total pressure applied to the entrance of the capillary. The values of η given in Table I were obtained by the use of Equation 18 which assumes that the capillary is filled with liquid.

T , °K.	\dot{m} g./sec.	ρ g./cc.	P_v	Pressures in dynes/sq. cm. P_h	P	η poises
4.225	1.3×10^{-9}	0.125	1.019×10^6	7×10^3	1.026×10^6
1.99	3.0×10^{-9}	.146	3.2×10^4	4×10^3	3.6×10^4	3.0×10^{-6}
1.90	6.2×10^{-9}	.146	2.4×10^4	4×10^3	2.8×10^4	1.1×10^{-6}
1.82	1.23×10^{-8}	.145	1.9×10^4	4×10^3	2.3×10^4	4.6×10^{-7}
1.77	1.76×10^{-8}	.145	1.6×10^4	4×10^3	2.0×10^4	2.8×10^{-7}
1.59	2.48×10^{-8}	.142	0.8×10^4	4×10^3	1.2×10^4	1.2×10^{-7}

TABLE II
DATA FOR CALCULATING THE VISCOSITY OF LIQUID HELIUM
Capillary B

In Table II P_1 refers to the average pressure at the exit of the tube and γ is the surface tension. P_γ is the pressure due to the action of surface tension in the tube.

T , °K.	\dot{m} g./sec.	ρ g./cc.	γ dynes/cm.	P_v	Pressure in dynes/sq. cm. P_h	P_1	P_γ
4.225	4.09×10^{-8}	0.125	0.098	1.019×10^6	7.4×10^3	4×10^3
2.983	7.45×10^{-9}	.141	.241	2.38×10^5	4.7×10^3	1.0×10^4
2.266	8.31×10^{-9}	.146	.311	6.17×10^4	4.1×10^3	1.2×10^4
2.266	8.11×10^{-9}	.146	.311	6.17×10^4	2.8×10^3	1.2×10^4
2.131	8.70×10^{-8}	.146	.323	4.57×10^4	2.6×10^3	1.3×10^4
2.020	1.61×10^{-7}	.146	.330	3.45×10^4	1.9×10^3	1×10^2	1.3×10^4
1.928	2.82×10^{-7}	.146	.336	2.65×10^4	1.6×10^3	2×10^2	1.3×10^4
1.884	3.52×10^{-7}	.146	.338	2.32×10^4	1.4×10^3	2×10^2	1.3×10^4
1.823	5.42×10^{-7}	.146	.342	1.90×10^4	1.1×10^3	3.2×10^2	1.4×10^4
1.725	5.17×10^{-7}	.145	.347	1.37×10^4	8.5×10^2	3.1×10^2	1.4×10^4
1.610	4.68×10^{-7}	.142	.351	8.36×10^3	7.1×10^2	2.9×10^2	1.4×10^4
1.468	4.53×10^{-7}	.142	.353	4.65×10^3	4.3×10^2	2.8×10^2	1.4×10^4

liquid helium ran up the thermocouple wire and was able to maintain itself despite the heat input by means of the eddy currents. At the conclu-

sion of the experiment the level was checked by evaporating and measuring the amount of the residual helium. Sufficient observations were available so that a curve showing the bath level as a function of time could be constructed.

The observations on capillaries A and B are given in Tables I and II. Other data necessary for the calculations are also tabulated.

The density of liquid helium has been determined at various temperatures by Boks and Kamerlingh Onnes.¹⁴ The values given in our tables are taken from curves through their data. Similarly the values of surface tension have been obtained from the work of Van Wik, Keesom and Kamerlingh Onnes¹⁵ and Allen and Misener.¹⁶

The flow data given in Table II are shown graphically in Fig. 1. The observations were made in one continuous series.

The evaluation of the tube constants was carried out by use of Equations 17 and 18. Van

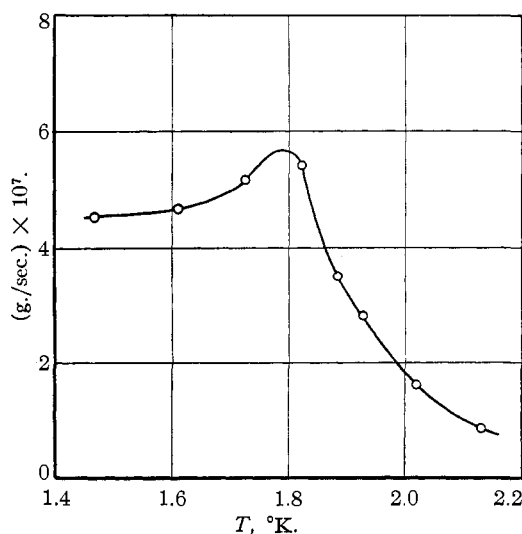


Fig. 1.—Amount of helium flowing through tube B. Liquid flow below 1.8°K. Gas and liquid flow above 1.8°K.

(14) Boks and Kamerlingh Onnes, *Comm. Phys. Lab. Leiden*, No. 170b.

(15) Van Wik, Keesom and Kamerlingh Onnes, *ibid.*, No. 179a.

(16) Allen and Misener, *Proc. Cambridge Phil. Soc.*, **34**, 299 (1938).

Itterbeek and Keesom¹⁷ have measured the viscosity of helium gas at the boiling point of helium. They give $\eta_g = 1.26 \times 10^{-5}$ poise at 4.23°K. It was necessary to know the value of η_l approximately and we have used the measurement of Keesom and MacWood.¹⁰ η_l was taken as 3.0×10^{-5} poise at 4.23°K. The value of λ in Equations 9-17 may be obtained from the measurements of Keesom and Kraak.¹⁸ We have estimated $\lambda = -2.5 \times 10^{-7}$ cm.² dyne⁻¹ at 4.225°K.

Let L_l = the length of tube filled with liquid and L_g the length filled with gas. For tube B $L_g/K = 1.38 \times 10^{16}$ cm.⁻³ and $L_l/K = 1.1 \times 10^{15}$ cm.⁻³. P_g was taken equal to P_v and $P_l = P_h + P_\gamma$, since the absence of appreciable heat leak below the surface of the liquid helium together with the rapid decrease in density with increasing temperature should make the temperature of the bath and tube approximate that of the surface. The surface tension pressure was evaluated with sufficient approximation assuming the annular type space, for which in this case $P_\gamma = 4\gamma/(D - d) = 3.9 \times 10^3$ dynes/cm.² at the boiling point. $L/K = L_g/K + L_l/K = 1.49 \times 10^{16}$ cm.⁻³. The fraction of the tube filled with liquid at 4.225°K. is found to be 7%. $L = 3.77$ cm. from which $K = 2.53 \times 10^{-16}$ cm.⁴. $K = \pi D(D - d)^3/96$ assuming an annular section. D was found to be 7.54×10^{-3} cm. by weighing a thread of mercury, from which $D - d = 1.01 \times 10^{-4}$ cm. $K = 5\pi D(D - d)^3/196$ for a crescent shaped section which leads to a value of 0.74×10^{-4} cm. for $D - d$.

From a similar treatment L/K for tube A was found to be 6×10^{17} cm.⁻³. $D = 0.05$ cm. $D - d$ was 8×10^{-6} cm. or 6×10^{-6} cm. for annular or crescent shaped section assumptions, respectively.

It is of some interest to compare the above values of $D - d$ for tube B with that calculated from thermal contraction. Assuming that the average difference between the coefficients of expansion of solder and Pyrex glass is 2.3×10^{-5} deg.⁻¹ for the temperature interval below the melting point of solder, $D - d$ is calculated to be 0.8×10^{-4} cm.

It will be convenient to divide the situations existing during the flow of helium through the tubes into two types as follows. Type I: Liquid and gas each occupying appreciable lengths of the

channel. $P_l = P_h + P_\gamma$, $P_g = P_v - P_i$. The flow is kept isothermal throughout by heat transfer through the tube wall. Type II: The liquid completely fills the tube and flows as a liquid into the line space. $P_l = P_h + P_v - P_i$. Again it is assumed that heat transfer keeps the flow isothermal.

The assumption as to isothermal flow of liquid in the cases given above is based on the fact that the channel is of extraordinarily small width and the velocity of flow very low. The maximum velocity of liquid flow was about 3 cm. sec.⁻¹.

The transition from Type I to Type II will occur when $P_\gamma > P_v - P_i$. The interpretation of experiments falling in the region of Type II is quite satisfactory.

In the case of tube A P_i was negligible and Type II flow should exist at all temperatures below that at which the vapor pressure $P_v = \frac{4\gamma}{D - d} = \frac{4 \times 0.35}{8 \times 10^{-6}} = 1.8 \times 10^5$ dynes/cm.² Since the highest vapor pressure in the experiments with Tube A was 3.2×10^4 dynes/cm.², corresponding to 1.99°K., it is evident that all observations except the one at 4.225°K. fall under Type II.

In the case of tube B the transition should occur when $P_v - P_i = P_\gamma = 4 \times 0.35/1.0 \times 10^{-4} = 1.4 \times 10^4$ dynes/cm.². As may be seen from Table II this would occur just above the measurement at 1.725°K. and when the results are calculated on the basis of Type I it is clearly evident that a transition does occur about where it would be expected. The transition, while it occurs rather abruptly, can hardly be expected to con-

TABLE III
VISCOSITY OF LIQUID HELIUM

T. °K.	η , poises, $\times 10^6$ assuming $\Delta P_l = P_h + P_\gamma$	η , poises, $\times 10^6$ assuming $\Delta P_l = P_v + P_h - P_i$
	Type I	Type II
2.983	35 ^a	..
2.266	20	..
2.266	19	..
2.131	1.8	5.4
2.020	0.91	2.2
1.928	.51	0.97
1.884	.40	.68
1.823	.27	.36
1.75	Calculated transition I \rightarrow II	
1.725	.28	.269
1.610	.30	.179
1.468	.30	.101

^a This method of determining the viscosity of the liquid becomes unreliable at higher temperatures where the fraction of the tube filled with liquid decreases. The value obtained at 2.983 is only approximate.

(17) Van Itterbeek and Keesom, *Physica*, 5, 257 (1938).

(18) Keesom and Kraak, *Physica*, 2, 37 (1935); *Comm. Phys. Lab. Leiden*, No. 234e.

form completely to the idealized conditions since the tube geometry is not likely to be exactly as assumed at the exit. The transition pressure calculated above assumed an annular channel and some displacement of the solder thread from a central position is to be expected. Moreover, the formation of the surface film mentioned above might be expected to play a part in the transfer of liquid near the exit. Nevertheless the expected transition does occur with considerable sharpness as may be seen from the results presented in Table III and shown graphically in Fig. 2 where the circles represent the results calculated on the assumption of Type I flow and the points the results calculated according to Type II flow.

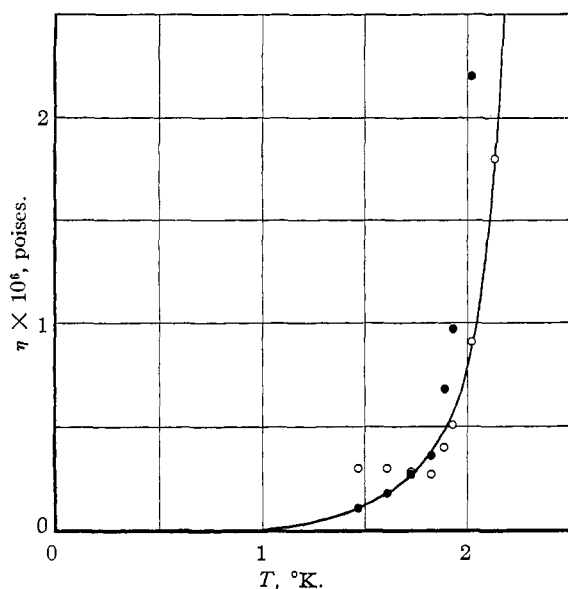


Fig. 2.—The viscosity of liquid helium: ●, assuming liquid throughout tube; ○, assuming liquid and gas in tube. The curve is the most probable value of viscosity. The type of flow changes near 1.8°K.

In calculating the results given in Table III the following considerations were used. For Type I flow the gas imperfection may be neglected below the "lambda point." Then

$$m = \frac{KM(P_v^2 - P_i^2)}{2RT(L - L_i)\eta_g} \quad (19)$$

also.

$$m = \frac{K\Delta P_i \rho_l}{L_i \eta_l}$$

Eliminating L_i from the above equations

$$\eta = \frac{(K/L)\Delta P_i \rho_l}{m - \frac{(K/L)M(P_v^2 - P_i^2)}{2RT\eta_g}} \quad (20)$$

It may be shown that the second term in the denominator of Equation 20 is negligible in the

region of helium II. In the most serious case it affects the viscosity by only 0.2%. Thus

$$\eta_i = \frac{K\Delta P_i \rho_l}{Lm}$$

for both Type I and Type II flow with

$$\Delta P_i = P_h + P_v \text{ and } \Delta P_i = P_v + P_h - P_i, \text{ respectively}$$

For the results above 2.2°K. the η_g of Equation 20 was obtained from a smooth curve through the data of van Itterbeek and Keesom.¹⁷

It is evident from both Figs. 1 and 2 that the transition occurs near 1.8°K. Above this temperature the interpretation is much less certain but it may be seen that the Type I points in Fig. 2 are low. This indicates some cooling at the liquid-gas interface when the liquid extends almost to the exit and may also indicate some effect of film formation around the exit. Cooling of the liquid surface allows P_v to contribute at least in part to the force acting on the liquid. The curve represents the most probable value of viscosity. All Type II points above 1.823°K. lie above the curve since $P_h + P_v$ would be expected to exceed the true P_i . However, the Type II point at 1.823°K. is low, probably because of film effect near the exit.

The largest value of Reynolds number $R = 2\bar{v}ph/\eta = m/\eta\pi D$ in the present experiments was 189 at 1.468°K. Davies and White¹⁹ have shown that R for the upper limit of laminar flow between parallel plates is 890.

TABLE IV
THE VISCOSITY OF LIQUID HELIUM
Values taken from a smooth curve through the data.

$T, ^\circ\text{K.}$	$\eta, \text{poises} \times 10^6$	$T, ^\circ\text{K.}$	$\eta, \text{poises} \times 10^6$
1.4	0.7	1.9	5.0
1.5	1.2	2.0	8
1.6	1.8	2.1	15
1.7	2.5	2.2	150
1.8	3.5		

As mentioned above, our results below 2°K. are lower than those of Keesom and MacWood by factors ranging from 10 to 22. We have no explanation to offer for this difference, which appears to be well outside the limits of observational error of each of the methods. In the case of the present work any error involved in obtaining the tube constants or interpreting the more reliable results which lie in the region below 1.8°K. would appear to make our viscosity values somewhat too high rather than too low.

(19) Davies and White, *Proc. Roy. Soc. (London)*, **A119**, 921 (1928).

Summary

The flow of liquid helium through small channels has been studied to obtain data desired in connection with the design of apparatus for adiabatic demagnetization experiments.

Formulas have been developed for the case of isothermal laminar flow when the material enters as a liquid and leaves as a gas.

The viscosity of liquid helium was found to de-

crease from 2.0×10^{-5} poise at 2.266°K . to 1.0×10^{-7} poise at 1.468°K . The largest Reynolds number, which occurred at 1.468°K ., was estimated to be about 200. At the lowest temperature of measurement the viscosity is still decreasing rapidly and appears to be approaching a value of zero at the absolute zero of temperature.

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[CONTRIBUTION FROM THE DEPARTMENT OF CHEMISTRY, LABORATORY OF PLANT NUTRITION, AND THE RADIATION LABORATORY, UNIVERSITY OF CALIFORNIA]

Radioactive Carbon in the Study of Photosynthesis

BY S. RUBEN, W. Z. HASSID AND M. D. KAMEN

In this preliminary report, there are presented some results obtained in the study of photosynthesis by the use of radioactive carbon, C^{11} , prepared in the Berkeley cyclotron. Experiments on barley plants were conducted under conditions of light and darkness as indicated below. Radioactive carbohydrates and chlorophyll (as phytochlorin and phytorhodin) formed in the living leaf on exposure to C^*O_2 were isolated.

Despite the 21.5 ± 0.5 minutes² half-life of C^{11} , the yields obtainable by bombarding amorphous boron³ with 8 MEV. deuterons are such as to make this method feasible. The nuclear reactions are ${}_1\text{D}^2 + {}_5\text{B}^{10} \rightarrow {}_6\text{C}^{11} + {}_0n^1$ and ${}_6\text{C}^{11} \rightarrow {}_6\text{B}^{11} + e^+$. The initial intensity was $10^7 - 10^8$ times the background of the Geiger counter used. The boron target was bombarded in a gas-tight chamber incorporating many features found to be of value in the Radiation Laboratory for the handling of high intensity beams.

The gas in the target holder was swept through a heated combustion tube containing cupric oxide into large Pyrex desiccators containing the plants. The bombarded boron was then introduced into the combustion tube and intensely heated in a stream of air. To facilitate the removal of the trapped radio-carbon a small piece of filter paper was placed in contact with the boron. The light source was a 500-watt floodlight directly above the desiccators and the temperature was kept at $28-30^\circ$.

The isolation of the water soluble radioactive carbohydrates was performed as follows. Immediately after removal from the C^*O_2 atmosphere, the plant was cut into small bits and immersed in boiling 80% ethanol for five

minutes.⁴ After filtration the extract was boiled to remove the alcohol. The radioactivity of the solution as measured by means of a small Geiger counter varied between 10^5 and 10^6 counts/min. at the time of measurement depending on the conditions of the experiment, the radioactivity of the solution being measured by pipetting out 1 or 2 cc. on a small piece of blotter (5×8 cm.) which then was partially dried over a hot-plate, covered with cellophane and wrapped around a small aluminum Geiger counter with 0.1 mm. wall thickness. This was found to be the most convenient and sensitive method. The following experiment was used to show that the activity was not caused by dissolved C^*O_2 . Sodium bicarbonate solution was added, followed by dilute acetic acid and the solution was then boiled vigorously. No detectable change in the radioactive content of the solution was produced.

Pigments as well as insoluble lead salts were then removed by the addition of charcoal and lead acetate, and filtration through talc. Thirty to forty per cent. of the C^* content of the solution was carried down by this precipitate.

Aliquots of the colorless filtrate were taken and osazones precipitated. Both the radioactivity of the simple reducing sugars (*i. e.*, those precipitable directly by phenylhydrazine) and that of the more complex carbohydrates (the osazone precipitation preceded by hydrolysis with boiling 1 *N* hydrochloric acid for three minutes) were measured. The crystalline osazone precipitates, after being carefully washed with water, dried and covered with cellophane, were wrapped around the Geiger counter. The radioactivity of the alcohol-water insoluble material, mainly cellulose, was determined by dissolving the plant residue in sulfuric acid.

The methods (Tswett absorption column, etc.) in use at the present time for a rapid isolation of chemically unaltered chlorophyll are not such as to preclude minute traces of colorless impurities. It was deemed necessary, therefore, to isolate a chlorophyll derivative.

(1) The symbol C^* will be used to represent carbon labeled by the radioactivity of some of its atoms.

(2) Determined in this Laboratory.

(3) We are indebted to Professor E. D. Eastman for the boron used in these experiments.

(4) From 90 to 95% of the water-soluble carbohydrates are removed by this procedure.